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Citation: American Journal of Physics **37**, 649 (1969); doi: 10.1119/1.1975735
View online: http://dx.doi.org/10.1119/1.1975735
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/37/6?ver=pdfcov
Published by the American Association of Physics Teachers

AMERICAN JOURNAL OF PHYSICS VOLUME 37, NUMBER 6 JUNE 1969

Undergraduate Experiment to Find Nuclear Sizes by Measuring Total Cross Sections for Fast Neutrons*

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(Received 24 October 1966; revision received 7 November 1968)

Pu–α–Be neutron sources, now available in many college laboratories, used with stilbene crystal detectors and proper circuits for neutron–gamma discrimination, permit students to measure fast-neutron total cross sections for a number of easily obtained samples. From these measurements they may calculate the size of nuclei and, using elements covering a wide range of the periodic table, demonstrate the constant density of nuclear matter.

INTRODUCTION

Very early in the history of nuclear physics, fast neutrons served as effective probes of nuclei. The lack of charge of the neutron greatly simplified the analysis of fast-neutron scattering. At energies on the order of 10 MeV, its effective size, 1.5×10^{-13} cm, given by its DeBroglie wavelength divided by 2π, is somewhat smaller than nuclear dimensions so that the scattering and absorption of neutrons by nuclei, is similar to the scattering and absorption of light in problems of optics.[1] The ease of an approximate interpretation resulting from this optical analogy enabled physicists to confirm estimates for the size and density of nuclei with fast-neutron, total cross-section measurements.[2]

Since one of the most characteristic properties of nuclear matter is its constant density, a simple laboratory experiment which permits students to establish the constant density of nuclei should be basic in an undergraduate introduction to nuclear physics. We believe the apparatus described in this report furnishes the opportunity for such a laboratory experiment at relatively minor expense.

As the footnotes referring to the student authors reveal, this experiment has evolved over a number of summer periods. A paper at a session of the Southeastern Section meeting of the American Physical Society[3] gave the essential details. One of us (Minor), using this paper for instructions, set up and ran the experiment with a minimum of outside assistance, and that mainly for assembling and adjusting the electronic equipment. We have revised the paper on the basis of this experience. We do not elaborate on the electronics, since the electronic details will depend on what commercial equipment is used. Unless otherwise specified, we present here only the detailed data from the last experiment.

I. APPARATUS

A Pu–α–Be source serves as the source of fast neutrons. In obtaining plutonium–beryllium neutron sources for instructional purposes, assistance is available to colleges and universities from the Division of Nuclear Education and Training, U. S. Atomic Energy Commission, Washington, D. C. 02545. For the results we report here, our Pu–α–Be source had a neutron intensity of 1.6×10^6 neutrons per second. The photograph in Fig. 1 shows one version of our experimental setup. We use the open geometry typical of many of the

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[1] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 324.

[2] R. Sherr, Phys. Rev. **68**, 240 (1945).

[3] F. D. Martin, H. E. Montgomery, Jr., L. M. Okun, and J. L. Fowler, Bull. Amer. Phys. Soc. **13**, 231 (1968).

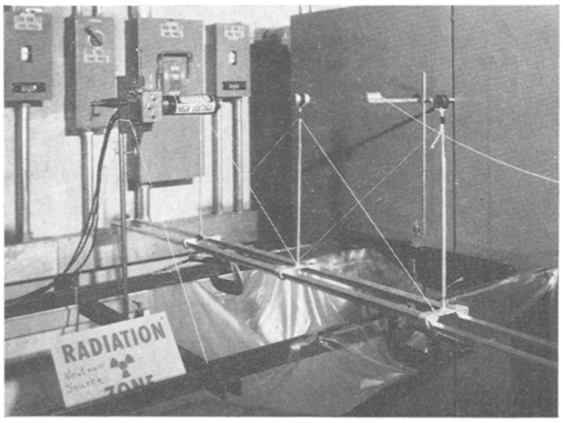

FIG. 1. Photograph of experimental apparatus for total neutron cross-section measurements.

published total cross-section experiments.[4] The Pu–α–Be source, here mocked up by an aluminum cylinder of the same diameter, rests on an aluminum support in a reproducible position. The support should be as light as stability permits to reduce the background of scattered neutrons to a minimum. The 1-in. diam$\times\frac{5}{8}$-in. long stilbene crystal, which detects fast neutrons by means of proton recoils, is supported about 56 cm in front of the source. The sample changer shown here allows one to move the sample between the source and the detector remotely, by means of cords attached to the support. A telescope with its cross hairs centered in the image of the sample enables one to align the samples between the centers of the source and detector. As can be seen, the sample support was put together from materials available around most physics laboratories. The samples are $1\frac{3}{8}$ in. in diameter.

Finding the neutron detector suitable for this experiment was of course the essential problem we had to solve. It had to satisfy several stringent conditions. In the first place, since the usable neutron flux from the available Pu–α–Be sources is limited, the neutron detector had to have high efficiency. In the second place, the detector circuits had to be able to discriminate against the gamma-ray background associated with the Pu–α–Be source. In order to select neutrons of reasonable energy definition, one uses only the upper end of the energy spectrum of the Pu–α–Be neutrons. Thus the bias level on the discriminator circuits must be arranged so as to eliminate low energy proton recoils in a reproducible manner.

We met the efficiency requirements by selecting an organic scintillator (the stilbene crystal) in which there is a rather high probability ($\sim$10%) that neutrons will produce proton recoils, of sufficiently high energy to give detectable photomultiplier pulses. The discovery that the light pulses from electrons in organic scintillators decay somewhat more rapidly than light pulses from proton recoils[5], makes possible circuits for discrimination against gamma-ray pulses in stilbene crystals. After trying out several of the circuits devised to discriminate against gamma rays,[6] we found one[7] sufficiently stable to allow us to obtain reproducible data for this experiment. The rapid increase in frequency of neutron pulses as bias settings are decreased, a consequence of the neutron spectrum from the Pu–α–Be source,[8] gives rise to the problem of stability. The circuit elements have to be stable enough to allow a constant small fraction of the neutrons to be detected and, at the same time, to discriminate effectively against gamma-ray pulses. As will be seen from the data to be presented, the pulse-shape discriminator described by Peelle and Love[7] adequately satisfies these requirements. We obtained the actual circuits used to take the data presented in this report from an instrument company.[9] The photomultiplier, stilbene crystal, and circuits required cost $\sim$\$6000. Of course, some of the circuits, i.e., the linear amplifier, would probably already be available for other uses around an undergraduate physics laboratory. If this experiment becomes generally accepted, surely this cost would decrease.[10]

[4] Dan W. Miller, *Fast Neutron Physics, Part II*, J. B. Marion and J. L. Fowler, Eds. (Wiley–Interscience, Inc., New York, 1963), p. 985.

[5] G. T. Wright, Proc. Phys. Soc. **69B**, 358 (1956).

[6] F. W. K. Firk, *Fast Neutron Physics, Part II*, J. B. Marion and J. L. Fowler, Eds. (Wiley–Interscience, Inc., New York, 1963), p. 2237.

[7] R. W. Peelle and T. A. Love, "Instruments and Techniques in Nuclear Pulse Analysis," National Academy of Sciences—National Research Council Publication No. 1184, 1964, p. 146.

[8] A. O. Hanson, *Fast Neutron Physics, Part I*, J. B. Marion and J. L. Fowler, Eds. (Wiley–Interscience, Inc., New York, 1960), p. 3.

[9] Oak Ridge Technical Enterprises Corporation.

[10] One of us (JLF) will be happy to supply a list of specifications for the detector and circuits required for this experiment.

II. EXPERIMENT AND RESULTS

After adjusting the circuits so that they discriminate against the gamma rays, as discussed by Peelle and Love,[7] the proton recoil bias has to be set to select a known energy segment from the upper end of the neutron spectrum. The published experimental and theoretical spectra of neutrons from Pu–α–Be sources[8] shows a maximum energy of $\sim$10.7 MeV, with an approximate linear increase of neutron intensity with decreasing neutron energy. Since this experiment concerns only the upper portion of the neutron spectrum, the problem is simplified by making a Taylor series expansion around the maximum neutron energy of the expressions required for deducing the average neutron energy from the bias curves. These include the neutron flux from the Pu–α–Be source $f(E)$, the (n,p) total cross section $\sigma(E)$, and the rate of change of scintillation light output with energy of recoil protons dp/dE. Thus,

$$f(E)=f_0+f_0'(E-E_0)+\cdots, \tag{1}$$

$$\sigma(E)=\sigma_0+\sigma_0'(E-E_0)+\cdots, \tag{2}$$

$$dP(E)/dE=P_0'+P_0''(E-E_0)+\cdots. \tag{3}$$

Since by definition $f(E)=0$ at $E=E_0$ in Eq. 1, $f_0=0$. Figure 6 (of Ref. 8) suggests the linear dependence of $f(E)$ on energy is an adequate representation of the source neutron flux from $E_0=10.7$ down to $\sim$7.5 MeV. The linear expressions[11,12] for $\sigma(E)$ and dP/dE are good to $\sim$1% over the energy interval of interest. For monoenergetic neutrons with energy less than $\sim$14 MeV, the energy distribution of recoil protons is uniform as indicated in Fig. 2.[13] The probability of counting a neutron with energy greater than the bias energy E_b is then proportional to the shaded area of the graph divided by the total area and also to the probability of scattering of the neutron by a proton in the crystal or $\sim[\sigma(E)\times(E-E_b)/E]$. With a spectrum of neutrons $f(E)$

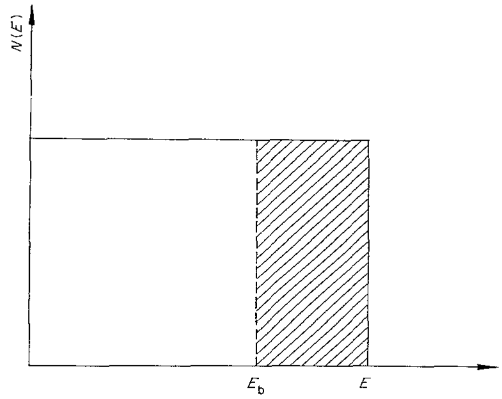

FIG. 2. Sketch of energy distribution of protons recoiling from neutron collisions.

with energies up to E_0, the total number of neutrons counted with a bias E_b is:

$$\eta=K\int_{E_b}^{E_0}[f(E)\sigma(E)(E-E_b)/E]dE, \tag{4}$$

where K is a constant of proportionality which depends on geometry, density of hydrogen atoms in the scintillator, etc. Using the expansions for $f(E)$ and $\sigma(E)$, and retaining terms in powers of $(E-E_0)=-\epsilon$ through ϵ^3, one obtains

$$\eta(E_b)=-K(f_0'\sigma_0/6E_0)\{1+\tfrac{1}{2}[(1/E_0)-(\sigma_0'/\sigma_0)]\epsilon_b^3\} \tag{5}$$

where $\epsilon_b=(E_0-E_b)$. Since the neutron flux and the (n,p) cross section decrease with increasing energy, f_0' and σ_0' are negative quantities and furthermore $-\sigma_0'/\sigma_0=0.084\ \mathrm{MeV}^{-1}$ (Ref. 11). Integrating Eq. (3), one has

$$P=P_0+P_0'(E-E_0)+(P_0''/2)(E-E_0)^2+\cdots, \tag{6}$$

or in terms of ϵ_b,

$$p=P_0-P_b=P_0'\epsilon_b[1-(P_0''/2P_0')\epsilon_b]. \tag{7}$$

From the data on p. 218 of Ref. 12, we find $P_0'=0.654$, and

$$P_0''/P_0'=0.027\ \mathrm{MeV}^{-1}.$$

Equations (7) and (5) give:

$$I(p)=-(Kf_0'\sigma_0/6E_0(P_0')^3)p^3\times\{1+[(1/E_0)-(\sigma_0'/\sigma_0)+(3P_0''/P_0')]p/2P_0'\}, \tag{8}$$

for the integral bias counting rate as a function of

[11] J. L. Gammel, *Fast Neutron Physics, Part II*, J. B. Marion and J. L. Fowler, Eds. (Wiley–Interscience, Inc., New York, 1963), p. 2185.

[12] C. D. Swartz and G. E. Owen, *Fast Neutron Physics, Part II*, J. B. Marion and J. L. Fowler, Eds. (Wiley–Interscience, Inc., New York, 1960), p. 211.

[13] J. L. Fowler and J. E. Brolley, Jr., Rev. Mod. Phys. **28**, 103 (1956).

$p = P_0 - P_b$. This becomes:

$$I(p) = Cp^3(1+0.20p) \qquad (9)$$

on substituting the numerical values of $1/E_0$, σ_0'/σ_0, and P_0''/P_0' into Eq. (8). $C = -[Kf_0'\sigma_0/6E_0(P_0')^3]$ is a positive constant.

In Fig. 3, closed triangles designate points calculated from experimental integral bias counting rates, by use of the relationship expressed in Eq. (9) in an iterative procedure. In first approximation, $I(p)$ is proportional to p^3, so that $[I(p)]^{1/3}$ is approximately proportional to p which by definition is linearly related to the pulse-height bias. The intercept of the curve $[I(p)]^{1/3}$ vs pulse height on the pulse-height axis then gives a first approximation to $p=0$, which by definition corresponds to $E_0 = 10.7$ MeV. Any other energy serves to determine the scale for p. In this particular case we used 8.3 MeV corresponding to $p = 1.51$, located, as described below, as the second energy. This energy together with $p=0$ found from the intercept, defines an approximate p scale. With values of p so determined, we calculate a correction factor $(1+0.20p)^{-1/3}$ for the ordinate of Fig. 2. According to Eq. (9) the corrected ordinate $[I(p)/(1+0.20p)]^{1/3}$ should be a linear function of p and therefore of the pulse-height bias. The straight line through the triangles in Fig. 3 indicates this is the case at least to $p \sim 2.2$ which corresponds to $E = 7.33$ MeV. The intercept on the pulse-height bias axis is slightly different (changes from 1960–1980) from that of the first approximation, and a second approximation for p can be made. The correction term $(1+0.20p)^{-1/3}$ is so insensitive to p, however, that no further interation is necessary.

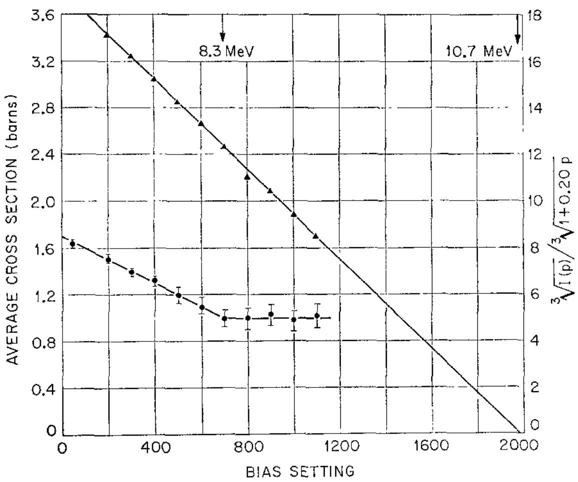

FIG. 3. Graphs for determining energy corresponding to bias settings. Closed triangles refer to right-hand ordinate, which is the function indicated of the integral counts, $I(p)$ at a given bias setting. Closed circles give uncorrected total cross section of carbon for neutrons of energy above the bias setting plotted as abscissa.

The total cross section of carbon shows an appreciable increase with decreasing energy near 8.3 MeV because of resonance structure.[14] Thus, the measured average cross section of carbon for neutrons of all energies above a given bias setting shows a break at the bias setting corresponding to 8.3 MeV. This break, clearly seen in curve through the solid circles in Fig. 3 together with the end-point energy E_0 establishes the neutron-energy scale. We describe how total cross sections are made below. The carbon cross sections in Fig. 3 have not been corrected for in-scattering.

Since the integrand of Eq. (4) gives the probability of counting a neutron of energy E when one uses a bias setting E_b, it is the weighting factor with which one can estimate the average energy of the neutrons producing the pulses above a given bias setting. Using the expansions for f, and σ in terms of ϵ, one calculates the average value of ϵ with the weighting factor

$$-(f_0'\sigma_0/E_0)\{1+[(1/E_0)-(\sigma_0'/\sigma_0)]\epsilon\}[\epsilon_b-\epsilon]\epsilon,$$

which is the integrand of Eq. (4) in terms of powers of ϵ. This factor is zero at $\epsilon=\epsilon_b$ and $\epsilon=0$, and has a maximum near $\epsilon_b/2$ so that the average energy is approximately one-half between the bias energy E_b and the end-point energy E_0. With a bias dial setting of 200, corresponding to 7.3 MeV, the average neutron energy is 8.94 MeV. To this we assign an uncertainty of ± 0.50 MeV arising not only from the uncertainty in the procedure for establishing the energy scale, but also from the lack of exact knowledge about the standards we use, such as, for example, E_0, the end-point energy for the Pu–α–Be neutrons.

The measurement of a neutron total cross section, one of the most basic of nuclear physics, involves a neutron-transmission experiment. The

[14] Donald J. Hughes and Robert B. Schwartz, Brookhaven National Laboratory, Upton, N. Y., *Neutron Cross Section*, 2nd ed., BNL–325 (1958).

TABLE I. Counting data and cross sections.

Sample	Observed counts per 40 min In	Observed counts per 40 min Out	Background per 40 min	Corrected counts per 40 min In	Corrected counts per 40 min Out	Uncorrected total cross section (barns)	Total cross section (barns)	$(\sigma_T/2\pi)^{1/2}$ cm $\times 10^{12}$	$A^{1/3}$ (amu)$^{1/3}$
Al	15 958±126	29 892±173	684±26	15 274±129	29 208±175	2.12±0.03	2.20±0.04	0.592±0.005	3.00
Fe	17 825±134	30 006±173	684±26	17 141±136	29 322±175	3.34±0.06	3.51±0.08	0.748±0.009	3.82
Cu	17 195±131	29 549±172	684±26	16 511±134	28 865±174	3.49±0.06	3.67±0.08	0.764±0.009	4.00
Sn	17 909±134	30 263±174	733±27	17 176±137	29 530±176	4.75±0.09	5.11±0.15	0.902±0.013	4.91
Pb	16 930±130	30 487±175	733±27	16 197±133	29 754±177	6.64±0.11	7.39±0.27	1.08±0.020	5.92

attenuation of neutrons by a sample in a geometry, like that shown in Fig. 1, is related to the nuclear cross section through a simple equation[4]

$$I_i/I_0 = \exp(-nt\sigma_T). \qquad (10)$$

Where I_i/I_0 is the ratio of counting rate with the sample in place to that with the sample out, n is the number of nuclei per cm^3 in the sample, t is the sample thickness, and σ_T is the neutron total cross section. In Table I the counts with the sample in and the counts with the sample out, for this experiment, are recorded in columns two and three, respectively. These are the combined counts for two runs. For each run we recorded counts for 20 min with the sample out and for 20 min with the sample in. By placing a $1\frac{3}{8}$-in. diam, 11.5 in.-long copper shadow bar between the source and the detector to remove direct neutrons, we found a correction to the "in" and "out" counts for background neutrons. After subtracting the background counts for the 40-min interval from both the "out" and the "in" counts, we used the data listed in the other columns together with the data of Table II to calculate the cross sections located in column 7.

Next we must make corrections for in-scattering—that is, for neutrons scattered forward into the detector by the sample. For the sample midway between the source and the detector, this correction is a minimum and is given by[4]:

$$\Delta\sigma_T = 4\pi(D/L)\sigma_n(0°), \qquad (11)$$

where D is the diameter of the sample, L is the distance between the source and the detector, and $\sigma_n(0°)$ is the differential cross section for forward angle scattering. For $\sigma_n(0°)$ we used Wick's limit[15] which gives an estimate of the differential-scattering cross section at 0° in terms of the total cross section

$$\sigma_n(0°) = (\sigma_T/4\pi\lambda\!\!\!^-)^2. \qquad (12)$$

At 14 MeV, Eq. (12) approximates the measured 0° differential cross sections of the nuclei we used to ~20%.[16] We have, therefore, combined a 30% uncertainty in the in-scattering correction in quadrature with the statistical errors to give the uncertainties listed in column 8 of Table I.

Simple qualitative arguments,[1] based on gemetric optics, lead to the expression:

$$\sigma_T = 2\pi(r+\lambda\!\!\!^-)^2, \qquad (13)$$

for the total cross section of a strongly absorbing spherical nucleus of radius r. $\lambda\!\!\!^-$, the neutron wave length divided by 2π, enters as the effective

TABLE II. Data on samples.

Sample	Length (cm)	Diameter (cm)	Mass (gm)	Density (gm/cm^3)
C	4.10	3.51	86.49	2.18
Al	5.07	3.49	131.47	2.71
Fe	1.91	3.49	143.29	7.84
Cu	1.92	3.50	162.92	8.82
Sn	3.09	3.50	215.99	7.26
Pb	2.82	3.49	304.18	11.30

[15] G. C. Wick, Atti. Acad. d'Ital. **13**, 1203 (1943). See also Ta-You Wu and Takashi Ohmura, *Quantum Theory of Scattering* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1962), p. 7.

[16] M. D. Goldberg, V. M. May, and John R. Stehn, Brookhaven National Laboratory, Upton, New York, *Angular Distributions in Neutron-Induced Reactions*, 2nd ed., Vol. II, BNL-400 (1962).

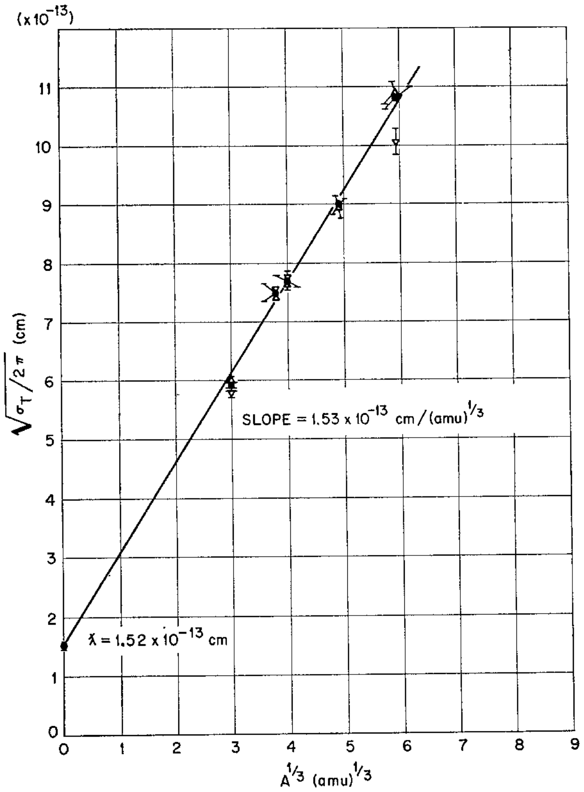

FIG. 4. The square root divided by 2π of cross sections plotted as function of $A^{1/3}$ of the sample. Closed circles represent data from Tables I and II. Open triangles give data from two previous experiments.

radius of the neutron. Thus if one takes the square root of the measured cross sections divided by 2π, one obtains an expression linear in the radius of the nucleus. In Fig. 4 we plot this expression $(\sigma_T/2\pi)^{1/2}$ as a function of $A^{1/3}$, the cube root of the number of nucleons in the nucleus, and make the line pass through the point 1.52×10^{-13} cm at $A^{1/3}=0$. The 3% error of this latter point is determined by the uncertainty of the neutron energy. The solid circles in Fig. 4 give the data from Table I. The triangles represent data from earlier versions of this experiment. The equation of the line in Fig. 3 is:

$$(\sigma_T/2\pi)^{1/2}=[1.53A^{1/3}+1.52]\times10^{-13}\text{ cm}, \tag{14}$$

where the constant 1.53×10^{-13} agrees with the value of this quantity obtained from other measurements.[17] Since the nuclear radius goes as $A^{1/3}$, Eq. (4) indicates nuclear density is constant, $\sim0.7\times10^{38}$ nucleons per cm^3.

After becoming familiar with circuits, including finding the bias settings, the entire experiment requires about 16 h.

ACKNOWLEDGEMENTS

We wish to acknowledge the advice and assistance we received from Dr. J. A. Biggerstaff and R. P. Cumby in selecting and setting up the detector for this experiment.

[17] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 13.