# Conversion Coefficients of the $Fe^{57}$ 14.4-keV Transition, the K-Capture Fraction in $Co^{57}$ $e^-$ Capture, and the K-Fluorescence Yield of Fe*

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By x-x and x-$\gamma$ coincidence-counting a $Co^{57}\rightarrow Fe^{57}$ source with gas proportional counters, we obtain the values $\alpha_K\omega_K=2.484$ ($\pm1.8\%$), $f\omega_K=0.3044$ ($\pm1.4\%$), and $S/(\alpha_T+1)=0.09482$ ($\pm2.3\%$), where $\alpha_K$ and $\alpha_T$ are the $K$ and total conversion coefficients of the $Fe^{57}$ 14.4-keV $\gamma$ ray, $\omega_K$ is the $K$-fluorescence yield of Fe, $S$ is the probability that the $Fe^{57}$ 136.4-keV level decays to the 14.4-keV level, and $f$ is the fraction of $K$ captures in $Co^{57}$ $e^-$ capture. These numbers are consistent within narrow limits with others' reported precise measurements of $S$, $f$, and the ratio $\alpha_K/\alpha_T$. With the use of the values $S=0.87$ ($\pm1.7\%$) and $\alpha_K/\alpha_T=0.89$ ($\pm0.15\%$) from electron-spectrometer measurements reported by others, we obtain $\alpha_T=8.17$ ($\pm3.1\%$); $\alpha_K=7.27$ ($\pm3.1\%$); $f=0.891$ ($\pm3.85\%$); $\omega_K=0.342$ ($\pm3.6\%$). Our value of $\alpha_T$ is 9% lower than that reported for some half-dozen measurements by $\gamma$-$\gamma$ coincidence counting and by Mössbauer methods, all of which cluster closely about the value 9.0. It closely confirms the theoretical $\alpha_T$. Our $f$ value agrees closely with one measured by multiwire proportional counter, and our $\omega_K$ agrees with a recent direct measurement.

## INTRODUCTION

THE internal conversion coefficient of the $Fe^{57}$ 14.4-keV $\gamma$ ray of Mössbauer fame has been the subject of numerous measurements by a variety of coincidence-counting and Mössbauer methods. The trend of the successive reported values of this coefficient is one of fairly steady decrease from an initial high of 15, levelling off in the last few years to a value of about 9. In the present work we report a yet lower value, 8.17 ($\pm3.1\%$), as one result of an alternative coincidence-counting method that simultaneously gives values of the $K$-fluorescence yield of Fe and the $K$-capture fraction in $Co^{57}$ $e^-$ capture.

In studies on the parent-daughter pair $Co^{57}\rightarrow Fe^{57}$ by coincidence-counting, it proves especially profitable to measure both the x-x and x-$\gamma$ rates of coincidence of Fe $K$ photons from $Co^{57}$ $K$ capture with, respectively, the Fe $K$ photons from internal conversion in $Fe^{57}$ and the $Fe^{57}$ 14.4-keV $\gamma$ ray. These two coincidence rates are functions of the probabilities entered in the decay scheme[1] in Fig. 1 and listed in Eqs. (8)—seven independent probabilities in all, including $\omega_K$—and one can derive many interlocking equations expressing various combinations of the probabilities in terms of the measured counting rates and efficiencies. In particular, with three such equations we find (see Eq. 42 below)

$$\alpha_K\omega_K=2.484\ (\pm1.8\%),$$

$$f\omega_K=0.3044\ (\pm1.4\%),$$

and

$$Sc_\gamma\equiv S/(\alpha_T+1)=0.09482\ (\pm2.3\%).$$

To extract the individual probabilities from these combinations, appeal must be made to the results of others. This can be done in several ways. We have chosen to use two results of low claimed error obtained by electron-spectrometer measurement. One of them, the value[2] $S=0.87$ ($\pm1.7\%$), inserted in the third of the preceding equations, gives the value of $\alpha_T$. This $\alpha_T$, together with the electron spectrometrically measured ratio[3] $\alpha_K/\alpha_T=0.89$ ($\pm0.15\%$), gives the value of $\alpha_K$, which, inserted in the first of the equations gives the value of $\omega_K$, which in turn, inserted in the second of the equations gives the value of $f$.

Thus, based on the most direct measurements—our photon counting and others' electron-spectrometer determinations of ratios of electron intensities—our method gives in one package the values

$$\alpha_T=8.17\ (\pm3.1\%),$$

$$\alpha_K=7.27\ (\pm3.1\%),$$

$$\omega_K=0.342\ (\pm3.6\%),$$

$$f=0.891\ (\pm3.85\%).$$

Our values of the $\alpha$'s and $\omega_K$ are, we believe, more accurate than previously published ones. Our value of $f$

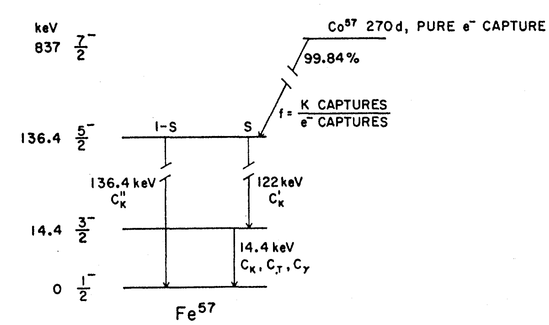


FIG. 1. Relevant part of the $Co^{57}\rightarrow Fe^{57}$ decay scheme. The symbols appended to the various transitions are defined in Eqs. (8).

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[1] For the complete decay scheme see (a) O. C. Kistner and A. W. Sunyar, Phys. Rev. 139, B295 (1965); or (b) G. D. Sprouse and S. S. Hanna, Nucl. Phys. 74, 177 (1965).
[2] D. C. Hall and R. G. Albridge, Nucl. Phys. A91, 495 (1967).
[3] J. B. Bellicard and A. Moussa, J. Phys. Radium 18, 115 (1957).

agrees closely with the result of an accurate measurement made by means of a multiwire proportional counter. Our value of $\omega_K$ is in line with the now recognized trend of values of fluorescence yields in this region of the atomic table (see Ref. 7). Our value of $\alpha_T$ closely confirms the theoretical value for a pure $M1$ transition. It is about 10% lower than the generally accepted value 9.0, but its correctness finds strong support from the fact that the equation from which it is obtained is one of three interlocking equations that are consistent within close limits with published values of $f$, $S$, and the ratio $\alpha_T/\alpha_K$, all of which have low claimed error.

## I. EXPERIMENTAL DETAILS

### A. Apparatus

The photon detectors are two almost identical cylindrical brass proportional counters, aluminum lined to eliminate fluorescent x rays from the brass, about 12 in. long and 3.5-in. i.d., filled at 24°C with 680.1 Torr of Ar plus 75.5 Torr of $CH_4$, and admitting photons through a $\sim$20-mg/cm² Be window sealed over a 1-in.-diam hole in a flat surface built up on the cylindrical counter wall. We remark parenthetically that though, as x-ray counters, proportional counters are now inferior to Li-drifted silicon detectors in almost every way, they still remain the basic detector for *absolute* measurements of x-ray intensities by virtue of the accuracy and ease with which their counting efficiencies can be determined.

The electronics are conventional. The coincidence time used was $2\tau=2.5$ μsec, as measured by counting random coincidences with independent sources. This long coincidence time, which is necessary with our proportional counters because the pulse start-up times vary over a range of 0.75 μsec after the initiating ionization act, does not constitute much of a disadvantage in the present case because a coincidence time of $\sim$1 μsec is necessary anyway in view of the 0.1-μsec half-life of the 14.4-keV level of $Fe^{57}$.

### B. $Co^{57}$ Source

This was prepared from a commercially supplied neutral solution of $Co^{57}Cl_2$. A droplet of the solution, about 2λ in volume and $\frac{1}{2}$-mm in diameter, was deposited accurately at the center of an 0.9-mg/cm² Mylar film 1.5 in. in diameter, stretched drum-tight on a flat aluminum ring. The droplet was allowed to evaporate at room temperature and then covered with another film of 0.9 mg/cm² Mylar. The resulting active spot had an x-photon emission rate of about $10^5$ per min and was invisible to the naked eye. A spectrum of the source covering energies up to 2 MeV, taken with a lithium-drifted germanium detector, showed no contaminant radiations other than the 1173- and 1332-keV γ rays of $Co^{60}$, with intensities corresponding to 0.02% of the $Co^{57}$ decays, as estimated by comparison with the intensity of the $Co^{57}$ 692-keV γ ray, which is known to be emitted in 0.14% of the decays.[1]

### C. Measurement of Counting Rates and Counting Efficiencies

In all of the following when we refer to "the γ ray" we mean the 14.4-keV γ ray; the other γ rays will be explicitly designated as the 122-keV γ ray and the 136.4-keV γ ray.

The measurements needed in our work are those of the x-x and x-γ coincidence rates, the x and the γ singles rates and their counting efficiencies in one of the counters, and the x singles rate and its counting efficiency in the other counter, all for the source in coincidence-counting position. In coincidence-counting position the source was located in an accurately reproducible position between the two counter windows, about 1 cm from counter No. 1 and 3 cm from counter No. 2. (Such relatively large source distances are advisable to reduce to manageable levels distortions in the single rates due to pulse-summing; see Appendix B.) Counter No. 1 was electronically channeled to register only the x rays, while counter No. 2 registered both x rays and γ rays, so that the x-x and x-γ coincidence rates were both obtained in the same run. Coincidence-pulse recording was in two-parameter mode, 64×64 channels. All singles counting was done by taking singles spectra, so the x- and γ-ray singles rates in counter No. 2 were taken simultaneously.

#### *1. Coincidence Count Rates*

The uncorrected x- and γ-ray spectrum registered by counter No. 2 in coincidence with the x-ray spectrum registered by counter No. 1, as obtained in a 69-h count, is shown in Fig. 2. Figure 3 shows the corresponding singles spectrum obtained with counter No. 2 in a 160-min count. The shapes of the two spectra differ only slightly. The x-ray peak of the coincidence spectrum is a little wider than that of the singles spectrum; the γ-ray peak of the coincidence spectrum is narrower than that of the singles spectrum because superposed on the latter is an 8% contribution of 12.8-keV sum pulses (see Appendix B). Table I shows the integrated x-x and x-γ coincidence rates, and the corrections. The correction for random coincidences was computed from the measured singles rates and the measured coincidence time $2\tau=2.5$ μsec. The contribution of the 122-keV and 136.4-keV γ rays to the coincidence background is the sum of the two coincidence rates obtained with the source in counting position and with the absorber over first one counter window and then the other. The absorber consisted of 109 mg/cm² of Cu sandwiched between 64.5 mg/cm² and 105 mg/cm² of Al, these thicknesses being selected to suppress the x-ray, 14.4-keV γ-ray, and Cu fluorescence radiations effectively and completely while absorbing only 1% of the 122-keV and 136.4-keV $\gamma$ rays.

Another conceivable correction—that for sum pulses arising from simultaneous emission of three photons by the source—can be shown to be negligible by a detailed analysis.

From Table I the corrected x-x and x-$\gamma$ coincidence rates, which we designate, respectively, by $r_{xx}$ and $r_{x\gamma}$, are

$$r_{xx}=105.56/\text{min} \quad (\pm 0.16\%),$$
$$r_{x\gamma}=5.226/\text{min} \quad (\pm 0.8\%), \qquad (1)$$

where the indicated error is the square root of the sum of the gross and background counts expressed as percent of the corrected coincidence count. This means that we claim that no errors other than those of counting statistics need be taken into account. Justification for this claim is given in the discussion of counting efficiencies below.

### 2. *Singles Count Rates*

The singles counts minus the backgrounds of the source in coincidence counting position were as follows:

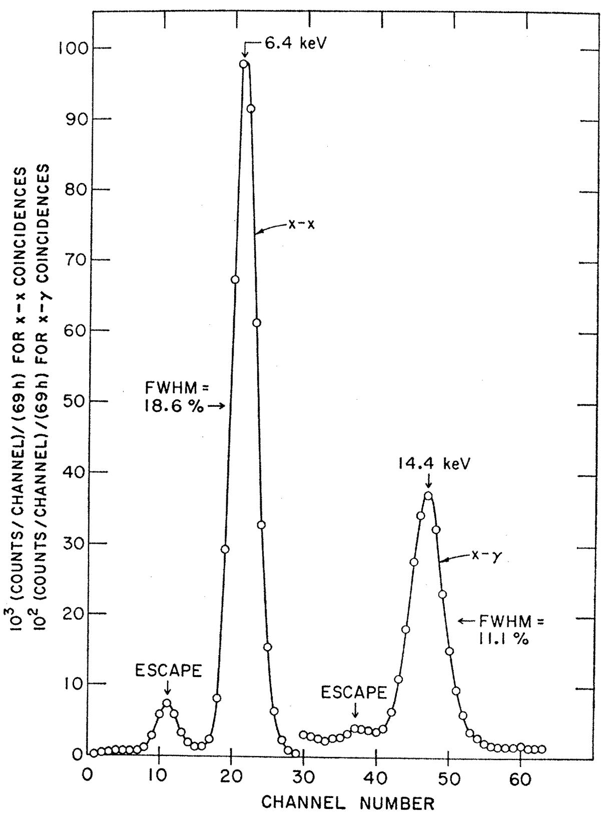


FIG. 2. Uncorrected spectrum of x and $\gamma$ rays in counter No. 2 in coincidence with the x-ray spectrum in counter No. 1, obtained in a 69-h count. The coincidences were taken in the two-parameter mode, and the spectrum shown is the result of integrating over the parameter on which the pulses from counter No. 1 were displayed.

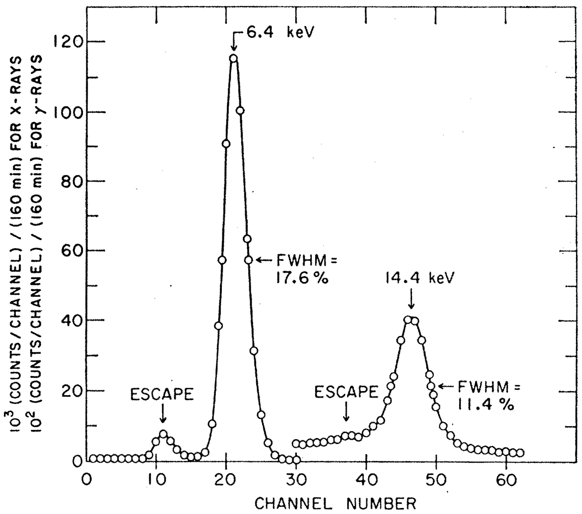


FIG. 3. Uncorrected singles spectrum observed in counter No. 2 with the source in coincidence-counting position. 160-min count.

In Counter No. 1:

$$\text{x-ray count}=128474-641 \quad =127833/10 \text{ min},$$

In Counter No. 2:

$$\begin{cases}\text{x-ray count}=503273-12414=490859/160 \text{ min},\\ \gamma\text{-ray count}= \ 35020-11390=23630/160 \text{ min}.\end{cases}$$

Backgrounds were taken with the source in counting position, the background in counter No. 1, which registers only x rays, being taken with 64.5 mg/cm$^2$ of Al over the window, and that in counter No. 2, which registers both x rays and $\gamma$ rays, with the window covered by 109 mg/cm$^2$ of Cu backed by 105 mg/cm$^2$ of Al. From the preceding values the corrected singles rates, after small decay corrections (1.5 days for the counter No. 1 results and 12.5 days for the counter No. 2 results) back to the time of the coincidence runs, are:

Counter No. 1:

$$\text{x-ray rate}=R_1=12834/\text{min} \quad (\pm 0.3\%),$$

Counter No. 2:

$$\begin{cases}\text{x-ray rate}=R_2= \ 3166.4/\text{min} \quad (\pm 0.15\%),\\ \gamma\text{-ray rate}=R_\gamma= \ 152.4/\text{min} \quad (\pm 0.9\%).\end{cases} \qquad (2)$$

TABLE I. Coincidence count rates.

| Coincidences gross rate | x-x 451175/(69 h) | x-$\gamma$ 26480/(69 h) |
|---|---|---|
| Corrections: | | |
| Natural bkgd. | 1484 | 1487 |
| Chance coinc. | 7077 | 507 |
| Due to other $\gamma$'s | 5610 | 2849 |
| Total corr. | 14171/(69 h) | 4843/(69 h) |
| Corrected rate | 437004/(69 h) | 21637/(69 h) |

Here, again, the error is the square root of the sum of the gross and background counts, expressed as percent of the corrected count. These singles rates require no correction for random sum pulses, but they do require corrections for true sum pulses due to absorption in the same counter of both photons of x-x and x-$\gamma$ pairs. This correction was made purely on the basis of our own counting data by the method described in Appendix B. The singles count rates so corrected, designated by lower case letters corresponding to the capital letters of the uncorrected rates in (2), are, with rounded-off errors:

$$r_1=13289/\text{min} \quad (\pm 0.3\%),$$
$$r_2=3191.6/\text{min} \quad (\pm 0.2\%), \qquad (3)$$
$$r_\gamma=141.0/\text{min} \quad (\pm 1.0\%).$$

### 3. *Absolute Emission Rates and Counting Efficiencies*

In order to measure these rates and efficiencies in a way that would provide a basis for error estimates we made use of a strong auxiliary source that gave convenient counting rates over a fairly wide range of counting efficiencies. The x-ray and $\gamma$-ray absolute emission rates of the auxiliary source were determined in four partially independent measurements, by singles counting in four different geometries for which the counting efficiencies could be calculated accurately from the relevant geometrical dimensions and x-ray attenuation coefficients.[4] The absolute emission rates of our primary source (i.e., the source used in the coincidence counting) were then obtained by comparison counting against the auxiliary source, and with these rates known the counting efficiencies in coincidence-counting position were obtained by use of the singles rates (3).

The results of the four measurements on the auxiliary source, details of which are given in Appendix A, are shown in Table II. Here the corrected count rates in columns 3 and 4 include corrections for sum pulses made by the method of Appendix B, and the indicated errors are solely those of counting statistics. In the computed efficiencies, columns 5 and 6, the errors are 0.1% for the x rays and 0.3% for the $\gamma$ rays (see Appendix A). These errors are due almost exclusively to errors in the x-ray attenuation coefficients that enter the efficiency calculations, the errors in the geometrical dimensions being negligible. We measured the x-ray attenuation coefficients ourselves and checked them against the literature values.

We digress here to discuss our counting errors. In the last two columns of Table II it is seen that the relative emission rates for the four source distances are the same within counting statistics. We take this as an *a posteriori* justification for assuming that the errors in our count rates are solely those of counting statistics, arguing as follows: Since the emission-rate ratios in Table II are relatively independent of errors in the x-ray attenuation coefficients, the doubtful point about this assumption is the accuracy of our background corrections. The background corrections to the x-ray rates were small, but for the $\gamma$-ray rates they varied from 10.2% of the corrected $\gamma$-ray count rate on shelf 2 to 50.7% of the corrected rate on shelf 5. If the error in the estimates of $\gamma$-ray background were 10% of the background, the ratio of the shelf 2 to shelf 5 $\gamma$-ray emission rates would be at least $1\pm0.05$—well outside the observed ratio. If the error in the background estimate were 5% of the background, the resulting shelf 2 to shelf 5 ratio of $\gamma$-ray rates would not be inconsistent with counting statistics. But the effect of a 5% error in the background would be to increase our listed error only slightly, so we disregard it.

To resume the main line of discussion, as the absolute emission rates of the auxiliary source we adopt the shelf-5 values in Table II, with errors obtained by compounding the counting statistics with the errors in the computed counting efficiencies. These absolute emission rates are

Auxiliary-source-emission rates:

$$\text{x ray:} \quad 1.866\times10^6/\text{min} \quad (\pm0.3\%),$$
$$\gamma \text{ ray:} \quad 3.217\times10^5/\text{min} \quad (\pm1.1\%). \qquad (4)$$

The ratio of strengths of the auxiliary and primary sources was obtained by comparison x-ray counting in the shelf-2 position. After all corrections of the types described previously, this ratio proved to be

$$\frac{\text{Auxiliary source strength}}{\text{Primary source strength}}=18.250 \ (\pm0.3\%). \qquad (5)$$

From (4) and (5), the absolute x-ray and $\gamma$-ray emission rates of the primary source, which we designate by $x_t$ and $\Gamma$, respectively, are, after correction for 33.5 days decay back to the mean time of the coincidence run

TABLE II. Absolute x-ray and $\gamma$-ray emission rates of auxiliary source.

| Shelf No. | Source distance (cm) | Corrected c/m x ray | Corrected c/m $\gamma$ ray | Computed efficiency x ray | Computed efficiency $\gamma$ ray | Absolute emissions/min x ray | Absolute emissions/min $\gamma$ ray | Relative emission rates x ray | Relative emission rates $\gamma$ ray |
|---|---|---|---|---|---|---|---|---|---|
| 2 | 2.521 | 21330 $\pm$0.25% | 1126.5$\pm$1.5% | $11.37\times10^{-3}$ | $35.32\times10^{-4}$ | $1.8760\times10^6$ | $3.1894\times10^5$ | 1.005 | 0.991 |
| 3 | 3.789 | 9352 $\pm$0.27% | 496.9$\pm$1.1% | $5.010\times10^{-3}$ | $16.00\times10^{-4}$ | $1.8667\times10^6$ | $3.1025\times10^5$ | 1.000 | 0.964 |
| 4 | 5.044 | 5149 $\pm$0.22% | 288.7$\pm$1.2% | $2.772\times10^{-3}$ | $9.092\times10^{-4}$ | $1.8575\times10^6$ | $3.1753\times10^5$ | 0.995 | 0.987 |
| 5 | 6.316 | 3219.5$\pm$0.18% | 186.9$\pm$1.0% | $1.725\times10^{-3}$ | $5.810\times10^{-4}$ | $1.8664\times10^6$ | $3.2169\times10^5$ | 1.000 | 1.000 |

[4] W. Rubinson and W. Bernstein, Phys. Rev. **86**, 545 (1952).

(correction factor = 1.090),

$$x_t = 1.1141\times10^5/\text{min}\quad(\pm0.4\%),$$
$$\Gamma = 1.921\times10^4/\text{min}\quad(\pm1.1\%). \tag{6}$$

From (3) and (6) the counting efficiencies of a source in coincidence-counting position are

$$\epsilon_1 = 0.1193\ (\pm0.5\%)\quad \text{(x rays in counter No. 1)},$$
$$\epsilon_2 = 0.02865(\pm0.5\%)\quad \text{(x rays in counter No. 2)}, \tag{7}$$
$$\epsilon_\gamma = 0.00735(\pm1.5\%)\quad (\gamma \text{ rays in counter No. 2}).$$

For convenience of reference we collect in Table III the values given in (6), (3), (7), and (1). These are the basic experimental values to be inserted in the equations that we proceed to derive.

## II. EQUATIONS

Since the derivations involve so many symbols and cross combinations, we provide detailed guidance through the maze.

The basic probabilities are (see Fig. 1)

$f$ = probability of $K$ capture in $Co^{57}$ $e^-$ capture decay,

$\omega_K$ = $K$-fluorescence yield of Fe,

$S$ = probability that the $Fe^{57}$ 136.4-keV level decays to the 14.4-keV level,

$c_T, c_K, c_\gamma$ = respectively, total-conversion, $K$-conversion, and $\gamma$-ray emission probability in the $Fe^{57}$ 14.4-keV transition,

$c_K', c_K''$ = respectively, $K$-conversion probabilities of the 122- and 136.4-keV $\gamma$ rays. (8)

In terms of the probabilities (8) we define

$\alpha_T = c_T/c_\gamma$ = total-conversion coefficient in the 14.4-keV transition,

$\alpha_K = c_K/c_\gamma$ = $K$-conversion coefficient in the 14.4-keV transition, (9)

$$k_1 = 1 + \frac{c_K'}{c_K} + \frac{(1-S)c_K''}{Sc_K}, \tag{10}$$

$$k_2 = 1 + \frac{c_K'}{f} = 1 + \frac{c_K'}{Sc_K}\frac{Sc_K}{f}, \tag{11}$$

$$k_3 = 1 + \frac{c_K'}{k_1 f} = 1 + \frac{k_2 - 1}{k_1}. \tag{12}$$

We use the following designations:

$N$ = $Co^{57}$ $e^-$-capture rate to the 136.4-keV level of $Fe^{57}$. (The 0.16% of the decays that take place to a higher level of $Fe^{57}$ do not affect our results.) (13a)

$x_1$ = emission rate of Fe $K$ photons from $Co^{57}$ $K$ capture, (13b)

TABLE III. Basic measured values.

| | | |
|---|---|---|
| Absolute emission rates: | | |
| | $x_t = 1.1141\times10^5$/min ($\pm0.4\%$) | |
| | $\Gamma = 1.921\times10^4$/min ($\pm1.1\%$) | |
| Singles counting rates: | | |
| | $r_1 = 13289$/min ($\pm0.3\%$) | (Counter No. 1) |
| | $r_2 = 3191.6$/min ($\pm0.2\%$) | (Counter No. 2) |
| | $r_\gamma = 141.0$/min ($\pm1.0\%$) | (Counter No. 2) |
| Counting efficiencies: | | |
| | $\epsilon_1 = 0.1193$ ($\pm0.5\%$) | (Counter No. 1) |
| | $\epsilon_2 = 0.02865$ ($\pm0.5\%$) | (Counter No. 2) |
| | $\epsilon_\gamma = 0.00734$ ($\pm1.5\%$) | (Counter No. 2) |
| Coincidence counting rates: | | |
| | $r_{xx} = 105.6$/min ($\pm0.16\%$) | |
| | $r_{x\gamma} = 5.226$/min ($\pm0.8\%$) | |

$x_2$ = emission rate of Fe $K$ photons from $K$ conversion of the $Fe^{57}$ 14.4-keV $\gamma$ ray, (13c)

$x_2'$ = emission rate of Fe $K$ photons from all $K$ conversion (i.e., including those of the 122-keV and 136.4-keV $\gamma$ rays of $Fe^{57}$), (13d)

$x_t = x_1 + x_2'$ = total emission rate of Fe $K$ photons, (13e)

and

$\Gamma$ = emission rate of 14.4-keV photons. (13f)

In terms of the probabilities (8), the emission rates in (13) are

$$x_1 = Nf\omega_K, \tag{14}$$

$$x_2 = NSc_K\omega_K, \tag{15}$$

$$\Gamma = NSc_\gamma, \tag{16}$$

and, with reference to Eqs. (10) and (15),

$$x_2' = N[Sc_K + Sc_K' + (1-S)c_K'']\omega_K = k_1 NSc_K\omega_K = k_1 x_2. \tag{17}$$

The rates (14)–(17) are singles emission rates, i.e., they are expressed in units of total number of photons/min, no distinction being made between a photon that is one of a pair and one that is single.

In deriving corresponding expressions for pair emission rates, account must be taken of the fact that a given kind of pair can be produced in several ways. Corrections to the pair rates due to the emission of x-x-x and x-x-$\gamma$ triples are trivial, as shown by a detailed analysis.

The emission rate $E_{xx}$ of x-x pairs is

$$E_{xx} = x_1(x_2'/N) + x_2 c_K'\omega_K \text{ (pairs/min)}$$

which, with use of Eqs. (17), (14), (15), and (12), can be written as

$$E_{xx} = x_1(x_2'/N)[1 + (Nc_K'\omega_K/k_1x_1)] = k_1x_1Sc_K\omega_K[1 + (c_K'/k_1f)] = k_1k_3x_1Sc_K\omega_K \tag{18}$$

and the emission rate $E_{x\gamma}$ of x-$\gamma$ pairs is

$$E_{x\gamma} = x_1(\Gamma/N) + \Gamma c_K'\omega_K \text{ (pairs/min)}$$

or, with the help of Eqs. (14), (16), and (11)

$$E_{x\gamma}=x_1(\Gamma/N)[1+(Nc_K'\omega_K/x_1)]$$
$$=x_1Sc_\gamma[1+(c_K'/f)]=k_2x_1Sc_\gamma. \quad (19)$$

The measured count rates are equal to the emission rates multipled by appropriate counting efficiencies.

The x-ray and $\gamma$-ray singles counting rates in counter No. 2, where the counting efficiencies are, respectively, $\epsilon_2$ and $\epsilon_\gamma$ are [see Eqs. (13e) and (13f), with Eqs. (14), (17), and (16)]

$$r_2=\epsilon_2 x_t, \quad (20)$$

$$r_\gamma=\epsilon_\gamma\Gamma, \quad (21)$$

where, by the definitions of $x_t$ and $\Gamma$, $r_2$ and $r_\gamma$ must be the observed singles count rates corrected for true sum pulses (see Appendix B).

In our measurement of coincidence count rates, counter No. 2 registered x rays and $\gamma$ rays with respective efficiencies $\epsilon_2$ and $\epsilon_\gamma$, and counter No. 1 registered x rays only, with efficiency $\epsilon_1$. Therefore the measured x-x and x-$\gamma$ coincidence rates $r_{xx}$ and $r_{x\gamma}$ are, in view of Eqs. (18) and (19) (and noting that an x-x pair has two ways of registering a coincidence),

$$r_{xx}=2E_{xx}\epsilon_1\epsilon_2=2k_1k_3x_1Sc_K\omega_K\epsilon_1\epsilon_2, \quad (22)$$

$$r_{x\gamma}=E_{x\gamma}\epsilon_1\epsilon_\gamma=k_2x_1Sc_\gamma\epsilon_1\epsilon_\gamma. \quad (23)$$

By means of these equations and Eqs. (20) and (21) we can express various combinations of the probabilites (8) in terms of our measured counting rates and efficiencies and the $k$'s.

Divide Eq. (22) by Eq. (23) to get

$$(c_K/c_\gamma)\omega_K\equiv\alpha_K\omega_K=(k_2/2k_1k_3)(\epsilon_\gamma/\epsilon_2)(r_{xx}/r_{x\gamma}). \quad (24)$$

This is the first of our principal equations. To get the second note that the combination $x_1Sc_\gamma\epsilon_\gamma$ in (23) can be written, in view of Eqs. (14), (16), and (21), as $f\omega_Kr_\gamma$, so we have from (23)

$$f\omega_K=r_{x\gamma}/k_2\epsilon_1r_\gamma. \quad (25)$$

Our third principal equation is Eq. (16) in the form

$$Sc_\gamma=\Gamma/N \quad (26)$$

with $N$ expressed, by use of Eqs. (14) and (13e), as

$$N=x_1/f\omega_K=(x_t/f\omega_K)[1-(x_2'/x_t)], \quad (27)$$

so that (26) becomes

$$Sc_\gamma=\frac{\Gamma}{x_t}\frac{f\omega_K}{[1-(x_2'/x_t)]}. \quad (28)$$

Here everything on the right-hand side except $x_2'$ is known in terms of the counting data and the $k$'s [see Eq. (25) and Table III], and $x_2'$ can be so expresesd as follows.

By Eqs. (17), (15), and (14),

$$x_2'=k_1x_2=k_1x_1(x_2/x_1)=k_1x_1Sc_K\omega_K/f\omega_K. \quad (29)$$

In Eq. (29) replace the numerator by its value from (22) and the denominator by its value from (25) to get the form sought,

$$x_2'=(k_2/2k_3\epsilon_2)r_\gamma(r_{xx}/r_{x\gamma}). \quad (30)$$

It will be useful in another connection to construct the reciprocal of the ratio $x_2'/x_t$ that appears in Eq. (28). From Eqs. (30) and (20),

$$x_t/x_2'=(2k_3/k_2)(r_2/r_\gamma)(r_{x\gamma}/r_{xx}). \quad (31)$$

Finally we derive an auxiliary equation that we will need later. From (29) and (13e)

$$Sc_K/f=x_2'/k_1x_1=x_2'/k_1(x_t-x_2')$$
$$=1/k_1[(x_t/x_2')-1]. \quad (32)$$

## III. NUMERICAL EVALUATIONS

Substitution of the basic measured values listed in Table III into our principal Eqs. (24), (25), and (28) gives

$$\alpha_K\omega_K=2.5885\frac{k_2}{k_3k_1}\ (\pm1.8\%), \quad (33a)$$

$$f\omega_K=\frac{0.3107}{k_2}\ (\pm1.4\%), \quad (33b)$$

$$Sc_\gamma=\frac{0.05348}{k_2[1-(x_2'/x_t)]}\ (\pm1.8\%), \quad (33c)$$

where the indicated errors do not include the errors in the $k$'s and $x_2'/x_t$.

The values of the $k$'s cannot be obtained from our own data. They all prove to exceed 1 by a few percent, and if we set them all equal to 1 we would incur errors of about 2%. They can be evaluated accurately by use of others' reported results, as follows.

Bellicard and Moussa[3] measured by electron spectrometer the ratio of the intensities of the $K$-conversion electrons from the 122-keV and 14.4-keV transitions (in our notation, $c_K'/c_K$) and the similar ratio from the 122-keV and 136.4-keV transitions [in our notation, $Sc_K'/(1-S)c_K''$]. From their reported results we compute

$$c_K'/c_K=0.023\ (\pm4.7\%),$$
$$(1-S)c_K''/Sc_K=0.02\ (\pm6.9\%), \quad (34)$$

where the indicated errors are computed from their claimed errors. The values (34) inserted into Eq. (10) give

$$k_1=1.043\ (\pm0.2\%). \quad (35)$$

To compute $k_2$ we use an iteration procedure that simultaneously gives a value of $x_t/x_2'$. In the right-hand form of (11) insert the value of $c_K'/c_K$ from

(34), the value of $S$ (which we shall use again later) from the electron-spectrometer measurement of Hall and Albridge,[2] namely

$$S=0.87\ (\pm1.7\%)\,, \tag{36}$$

and the expression for $Sc_K/f$ from Eq. (32). The result is, with $k_1$ from (35) ,

$$k_2=1+\frac{0.02535}{[(x_t/x_2')-1]}\,. \tag{37}$$

(Note that this procedure obviates knowledge of $f$.) Another relation between $k_2$ and $x_t/x_2'$ can be obtained by inserting the appropriate values from Table III into Eq. (31). With use of the second form of Eq. (12), this gives

$$x_t/x_2'=2.2404(k_3/k_2)=(2.2404/k_1)+(2.2404/k_2)[(k_1-1)/k_1]$$

or, with $k_1$ from (35) ,

$$x_t/x_2'=2.1480+(0.0923/k_2)\,. \tag{38}$$

Equations (37) and (38) are solved for $k_2$ and $x_t/x_2'$ by iteration, starting with $k_2=1$ in Eq. (38). The results are

$$k_2=1.0205\,, \tag{39}$$

$$x_t/x_2'=2.2387\ (\pm1.7\%)\,. \tag{40}$$

Then with the values (35) and (39), Eq. (12) gives

$$k_3=1.0197\,. \tag{41}$$

The errors in $k_2$ and $k_3$ are a few tenths of a percent, and so are negligible compared to the other errors. We remark that if the preceding analysis is carried through with the values (34) replaced by the corresponding values from the work of Hall and Albridge[2] the resulting values of the $k$'s and $x_t/x_2'$ differ from the preceding by 0.3%.

Insertion of the values (35), (39), (40), and (41) into Eqs. (33) gives

$$\alpha_K\omega_K=2.4837\ (\pm1.8\%)\,, \tag{42a}$$

$$f\omega_K=0.30444\ (\pm1.4\%)\,, \tag{42b}$$

$$Sc_\gamma\equiv S/(1+\alpha_T)=0.094821\ (\pm2.3\%)\,. \tag{42c}$$

These are the numbers obtained essentially from our own data. From them we extract the numbers of interest as follows.

With the Hall and Albridge value of $S$ stated in (36), Eq. (42c) gives

$$c_\gamma\equiv1/(1+\alpha_T)=0.10905\ (\pm2.8\%) \tag{43}$$

or

$$\alpha_T=8.17\ (\pm3.1\%)\,. \tag{44}$$

Thus we obtain $\alpha_T$ with the help of one published value. To go further we make use of Bellicard and Moussa's[3] electron-spectrometer measurements of the ratios of conversion electrons from the $K$, $L$, and $M$ shells in the $Fe^{57}$ 14.4-keV transition. They report (with our notation)

$$c_K/c_L=8.93\ (\pm1.6\%)\,,\quad c_L/c_M=9.1\ (\pm5\%)$$

from which we compute

$$c_K/c_T\equiv\alpha_K/\alpha_T=0.89\ (\pm0.15\%)\,, \tag{45}$$

a value almost identical with the value 0.892 (±0.54%) that we compute from corresponding data of Hall and Albridge.[2]

From (45) and (44),

$$\alpha_K=7.27\ (\pm3.1\%)\,. \tag{46}$$

From (46) and (42a),

$$\omega_K=0.342\ (\pm3.6\%)\,. \tag{47}$$

From (47) and (42b),

$$f=0.891\ (\pm3.85\%)\,. \tag{48}$$

To recapitulate, with the help of the two values (36) and (45) from electron-spectrometer work, we deduce from our counting data the values

$$\begin{aligned}\alpha_T&=8.17\ (\pm3.1\%)\,,\\ \alpha_K&=7.27\ (\pm3.1\%)\,,\\ \omega_K&=0.342\ (\pm3.6\%)\,,\\ f&=0.891\ (\pm3.85\%)\,,\end{aligned} \tag{49}$$

[assuming $S=0.87$ (±1.7%) and $\alpha_K/\alpha_T=0.89$ (±0.15%)],

where the errors are standard deviations and we believe there are no significant systematic errors.

We remark that our value of $\omega_K$ agrees with one of the most recently published values,[5] $\omega_K=0.347$ (±6.3%), and one of the oldest,[6] $\omega_K=0.343$. If we had *a priori* justification[7] for adopting, say, the first of these values, we could insert it into Eqs. (42a) and (42b) to get values of both $\alpha_K$ and $f$, then from the $\alpha_K$ and the value (45) get $\alpha_T$, which, inserted into (42c), would give the value of $S$. The results of this procedure are

$$\begin{aligned}f&=0.877\ (\pm6.5\%)\,,\\ \alpha_K&=7.16\ (\pm6.5\%)\,,\\ \alpha_T&=8.04\ (\pm6.5\%)\,,\\ S&=0.857\ (\pm6.2\%)\,,\end{aligned} \tag{50}$$

[assuming $\omega_K=0.347$ (±6.3%) and $\alpha_K/\alpha_T=0.89$ (±0.15%)].

[5] L. E. Bailey and J. B. Swedlund, Phys. Rev. **158**, 6 (1967).
[6] H. Lay, Z. Physik **91**, 533 (1934).
[7] Values of $\omega_K$ between 0.308±0.015 and 0.375 are tabulated by R. W. Fink, R. C. Jopson, H. Mark, and C. D. Swift, Rev. Mod. Phys. **38**, 513 (1966). The value given in the "Tables" of Wapstra *et al.* (see Ref. 9) is 0.293±0.005. Within the last year or two recognition has grown that the $\omega_K$ values of the lower $Z$ elements in these tables are low by about 15%.

Another alternative would be to use in Eqs. (42) a precise value of $f$ derived from the $Co^{57}$ $L/K$ capture ratio measured with a multiwire proportional counter by Moler and Fink.[8] They report the $L/K$ capture ratio$=0.099\pm0.011$. This number, together with the theoretical[9] $(M+N)/L$ capture ratio 0.092, gives

$$f=0.903\ (\pm1\%)\,. \qquad (51)$$

With this $f$ and the value of $\alpha_K/\alpha_T$ given in (45), Eqs. (42) give

$$\begin{aligned}\omega_K&=0.337\ (\pm1.7\%)\,,\\ \alpha_K&=7.37\ (\pm2.5\%)\,,\\ \alpha_T&=8.28\ (\pm2.5\%)\,,\\ S&=0.880\ (\pm3.2\%)\end{aligned} \qquad (52)$$

[assuming $f=0.903\ (\pm1\%)$ and

$$\alpha_K/\alpha_T=0.89\ (\pm0.15\%)].$$

The errors in the set (52) are actually somewhat smaller than those in the set (49). However, we have a slight preference for the numbers of the set (49); they involve, in a sense, fewer assumptions, since the results of others that we have used to obtain them (and the small corrections due to the $k$ values) are exclusively from one kind of measurement, viz., electron spectrometer.

## IV. DISCUSSION

The values (42), which, except for $\sim2\%$ corrections due to the $k$'s, derive exclusively from our own measurements, and the $S$, $f$, $\omega_K$, and $\alpha_K/\alpha_T$ values of others find mutual support in the consistency of the three sets of values (49), (50), and (52). A test of the consistency of the values (42) with the values of $S$, $f$, and the ratio $\alpha_K/\alpha_T$ is the following. Write Eq. (42c) in the form

$$\alpha_T=S/0.094821-1$$

and multiply by the ratio $f/\alpha_K$ from (42a) and (42b) to get

$$(f/S)(\alpha_T/\alpha_K)=1.29271[1-(0.094821/S)].$$

Insert into this equation the values of $f$, $S$, and the ratio $\alpha_T/\alpha_K$ from Eqs. (51), (36), and (45). The result is

$$1.1662\ (\pm2\%)=1.1517\ (\pm3.5\%)\,,$$

demonstrating consistency to within 1.25%.

There are two further pieces of work that support our results. Muir *et al.*[10] measured $x_t/\Gamma=5.58\ (\pm5.4\%)$, which is 0.962 of our $x_t/\Gamma$. Hall and Albridge[2] measured by electron spectrometer the ratio $e_K/e_{KA}=0.671$ $(\pm8.5\%)$ of 14.4-keV $K$ conversions to total $K$ Auger electrons, which value, together with our values of $x_t/\Gamma$ and $\omega_K$, gives $\alpha_K=7.49\ (\pm10\%)$, in agreement with our $\alpha_K$.

In view of this all-around consistency between different kinds of measurements we feel it highly unlikely that the values (49) can be off by much more than the indicated errors.

Measurements of $\alpha_T$ that do not depend on x-ray counting lie consistently about 10% higher than our value[11–14]:

| $\alpha_T$ | Method | Ref. |
|---|---|---|
| 9.0 (±0.5) | γ-γ coincidence | 1(a) |
| 9.94 (±0.6) | γ-γ coincidence | 11 |
| 8.9 (±0.6) | Mössbauer absorption | 1(a) |
| 9.0 (±0.4) | Mössbauer absorption | 12 |
| 8.9 (±0.7) | Mössbauer absorption | 13 |
| 9.2 (±0.5) | Mössbauer scattering | 14 |

In view of the magnitudes of the claimed errors, the individual values listed here, except for the 9.94, are not inconsistent with our $\alpha_T=8.17\ (\pm0.25)$. However, their mean, with neglect of the 9.94, is 9.0 (±0.12), which is inconsistent with our value. We remark that the first of the listed $\alpha_T$ values depends on the $K$-fluorescence yield of Rb, for which the authors took the value 0.629 from the tables of Wapstra *et al.*[9] This value appears low if one inspects a graph of measured $\omega_K$ values (see the compilation of Fink *et al.*[7]) versus $Z$. The not unlikely value of 0.67 for the Rb $\omega_K$ entails $\alpha_T=8.4$.

The $\alpha_T$'s from the method of Mössbauer absorption are obtained through

$$\alpha_T=23.60\times10^{-18}\ \text{cm}^2/\sigma_0-1\,,$$

where $\sigma_0$, the absorption cross section at resonance, is obtained from a theoretical value of the Mössbauer fraction $f_a$ of the absorber and a measured value of the product $f_a\sigma_0$. Starting with our $\alpha_T$ value and working this procedure backwards, we obtain $f_a$ values some 6% lower than the theoretical ones, but with overlapping errors.

Theoretical estimates of the conversion coefficients of the 14.4-keV transition are closely confirmed by our values. Assuming that the transition is pure $M1$, we obtained theoretical $\alpha_K$ and $\alpha_T$ values from an energy extrapolation of Rose's[15] values and also from a $Z$ extrapolation of the values of Hager and Seltzer,[16] with the results

| $\alpha_K$ | $\alpha_T$ | |
|---|---|---|
| 7.20 | 8.17 | (Rose) |
| 7.25 | 8.12 | (H. and S.) |

[8] R. B. Moler and R. W. Fink, Phys. Rev. **131**, 821 (1963).
[9] A. H. Wapstra, G. J. Nijgh, and R. Van Lieshout, *Nuclear Spectroscopy Tables* (Interscience Publishers, Inc., New York, 1959).
[10] A. H. Muir, Jr., E. Kankeleit, and F. Boehm, Phys. Letters 5, 161 (1963).
[11] H. C. Thomas, C. F. Griffin, W. E. Phillips, and E. C. Davis, Jr., Nucl. Phys. **44**, 268 (1963).
[12] R. H. Nussbaum and R. M. Housley, Nucl. Phys. **68**, 145 (1965).
[13] S. S. Hanna and R. S. Preston, Phys. Rev. **139**, A722 (1965).
[14] G. R. Isaak and U. Isaak, Phys. Letters **17**, 51 (1965).
[15] M. E. Rose, *Internal Conversion Coefficients* (Interscience Publishers, Inc., New York, 1958).
[16] R. S. Hager and E. C. Seltzer, California Institute of Technology Report No. CALT-63-60, 1967 (unpublished).

These are to be compared with our $\alpha_T$ and $\alpha_K$ values obtained without use of the $\alpha_K/\alpha_T$ ratio, namely, $\alpha_T=8.17$ ($\pm3.1\%$) as given in (49), and $\alpha_K=7.37$ ($\pm2.5\%$) as given in (52).

If we assume that $\alpha_T=8.17+3.1\%=8.42$, then an upper limit to the $E2/M1$ mixing ratio in the 14.4-keV transition is $\sim5\times10^{-4}$. A limit of $10^{-4}$ has been set by studies of conversion ratios in the $L$ subshells.[17]

Finally, we note that our $f$ value, which agrees with that from the work of Moler and Fink[8] provides another experimental support for Bahcall's exchange correction[18] in the theoretical calculation of $e^-$-capture probabilities.

## APPENDIX A: EXPERIMENTAL DETAILS ON THE DETERMINATION OF ABSOLUTE EMISSION RATES AND COUNTING EFFICIENCIES (See Sec. I C3)

### 1. *Calculation of the Counting Efficiencies at the Different Shelf Positions*

The counting efficiencies were calculated in the usual way from

$$\epsilon=(\Omega/4\pi)\exp(-\mu_at_a-\mu_bt_b-\mu_mt_m)(1-e^{-\mu_gt_g}),$$

where the $\mu$'s are mass attenuation coefficients for x rays, the $t$'s are absorber thicknesses averaged over angle, and the subscripts $a,b,m$, and $g$ refer, respectively, to air, Be, Mylar, and counter gas. $\Omega$ is the solid angle computed for the various distances (see Table II) of a point source from a circular aperture of 1.1974 cm diameter. The errors in $\epsilon$ due to errors in $\Omega$ and the $t$'s are negligible compared to those due to errors in the $\mu$'s.

Values of the $\mu$'s read from different published tabulations sometimes differ considerably and have no generally accepted error estimates. Therefore we measured the necessary $\mu$'s ourselves by taking the spectrum of a strong $Co^{57}$ source through different thicknesses of absorber. The values we found are given in Table IV, where for comparison we have also entered, in parentheses, values obtained by interpolation in published tabulations. Our Be value for x rays was obtained with the use of Be foils that, like the Be of the counter windows, were known to be of high purity by spectrographic analysis at this laboratory. It is based on only three points covering a range of $\frac{1}{3}$ of a half-thickness, but it serves to confirm the lower of the two published values that differ by 50%. In view of their consistency with the literature values we assume that the extreme errors in the air and Be $\mu$ values for x rays are $\pm2\%$. Errors in the corresponding values for the 14.4-keV $\gamma$ rays are unimportant because of the trivial amount of $\gamma$-ray absorption.

TABLE IV. Photon attenuation coefficients.

| Absorber \ Radiations | Fe $K$ spectrum | 14.4-keV $\gamma$ rays |
|---|---|---|
| Be | 2.15 cm²/g<br>(2.11[a])<br>(3.05[b]) | (0.33 cm²/g)[a] |
| Air | 18.62<br>(19.2[b]) | 1.82<br>(1.7[b]) |
| Ar | 206.3±1.6<br>(210.0[b])<br>(205.2[c]) | 22.5±0.3<br>(22.5[b])<br>(22.5[c]) |

[a] R. T. McGinnies, Natl. Bur. Std. (U. S.), Circ. 583, Suppl. (1959).
[b] A. H. Compton and S. K. Allison, *X Rays in Theory and Experiment* (D. Van Nostrand Co., Inc., New York, 1935).
[c] C. S. Barrett, *Structure of Metals* (McGraw-Hill Book Co., New York, 1943).

The errors in our Ar values are extreme errors estimated from semilog plots of count rate versus Ar pressure for 11 points extending over 7 half-thicknesses for the x rays and 1 half-thickness for the $\gamma$ rays.

In computing the errors in the absorption factor due to errors in the $\mu$'s, it was assumed that the magnitudes of the latter errors were $\frac{1}{3}$ the extreme error in order to make them correspond to standard deviations. The corresponding erorrs due to errors in absorber thicknesses were negligible. The total error in the absorption factors was 0.1% for the x rays and 0.3% for the $\gamma$ rays.

### 2. *Counting of the Auxiliary Source*

In counting position the source saw the counter window through a 1.1974-cm-diam aperture in a screen consisting of 1.36 g/cm² of Pb backed by successive screens of Cd, Cu, and Al, whose respective thicknesses were 0.69, 0.36, and 0.22 g/cm², each screen serving to suppress the fluorescence x rays excited in its predecessor. The aperture in the Cd-Cu-Al stack was 2 mm greater than that in the Pb, so it was the latter that defined the geometry. The material of the array of screens was completely opaque to the x rays and 14.4-keV $\gamma$ rays, and passed less than 0.5% of the 122-keV $\gamma$ rays and about 2% of the 136.4-keV $\gamma$ rays.

Counting times were long enough to accumulate gross counts of at least 150 000 for the x rays and 10 000 for the $\gamma$ rays.

Backgrounds were taken with the source in counting position and the aperture closed by an absorber consisting of 109 mg/cm² of Cu backed by 106 mg/cm² of Al (to absorb out the Cu fluorescence rays). This reduced the x-ray and 14.4-keV $\gamma$-ray intensities effectively to zero without much reducing the intensities of the 122- and 136.4-keV $\gamma$ rays, so that background contributions due to the presence of the latter were included in the background count. For the x rays the total background correction varied from 0.5% of the corrected rate at closest geometry to 2.8% at farthest. For the 14.4-keV $\gamma$ rays, the corresponding values were 10.2 and 50.7%.

[17] G. T. Ewan, R. L. Graham, and J. S. Geiger, Nucl. Phys. 19, 221 (1960)
[18] J. N. Bahcall, Phys. Rev. 132, 362 (1963).

Count rate corrections other than background were small. No correction was necessary for random sum pulses as shown by calculations based on a measured 0.95-$\mu$sec pulse time of the counter. Correction for the true sum pulses were made by the method described in Appendix B.

## APPENDIX B: CORRECTIONS FOR SUM PULSES

The absorption of both photons of an x-x or x-$\gamma$ pair in the same counter gives rise to a pulse-height corresponding to the sum of the photon energies, namely, 6.4+6.4=12.8 keV for x-x pairs and 6.4+14.4=20.8 keV for x-$\gamma$ pairs. Each 12.8-keV pulse constitutes a loss of two counts from the x-ray singles rate and, since our counters cannot resolve a 12.8-keV peak from the 14.4-keV peak and its escape peak at 11.4 keV, the spurious addition of one count to the $\gamma$-ray singles rate. A 20.8-keV pulse constitutes the loss of one count from both the x-ray and the $\gamma$-ray singles rates. Hence, if $R_x$ and $R_\gamma$ are the observed singles rates, and $R_{x+x}$ and $R_{x+\gamma}$ are the sum-pulse rates, then the corrected singles rates, $r_x$ and $r_\gamma$, are given by

$$r_x = R_x + 2R_{x+x} + R_{x+\gamma}, \tag{B1}$$

$$r_\gamma = R_\gamma - R_{x+x} + R_{x+\gamma}. \tag{B2}$$

If $\epsilon_x'$ and $\epsilon_\gamma'$ are the x and $\gamma$ counting efficiencies of the counter and if $x_1'$ is the emission rate of $K$-capture x rays from the source, then by analogy with Eqs. (22) and (23) of Sec. II,

$$R_{x+x} = k_1 k_3 x_1' S c_K \omega_K \epsilon_x'^2, \tag{B3}$$

$$R_{x+\gamma} = k_2 x_1' S c_\gamma \omega_k \epsilon_x' \epsilon_\gamma'. \tag{B4}$$

Note that Eq. (B3) lacks the factor 2 that appears in (22) because both x photons must be absorbed in the same counter.

In (B3) substitute the value of $k_1 k_3 S c_K \omega_K$ from (22), and in (B4) substitute the value of $k_2 S c_\gamma \omega_K$ from (23), to get, with use of (14),

$$R_{x+x} = (r_{xx}/2)(N'/N)(\epsilon_x'^2/\epsilon_1\epsilon_2), \tag{B5}$$

$$R_{x+\gamma} = r_{x\gamma}(N'/N)(\epsilon_x'\epsilon_\gamma'/\epsilon_1\epsilon_\gamma), \tag{B6}$$

where $N'$ is the strength of the given source at any known time, and $N$ is the strength of the primary source at the time of measuring $r_{xx}$ and $r_{x\gamma}$.

In applying (B5) and (B6) to correct the singles rates of the primary source in coincidence-counting position we have that $N'/N$ equals the decay factor for the time elaspsed between the coincidence counting and the singles counting, and that $\epsilon_x'$, $\epsilon_\gamma'$ are equal to $\epsilon_2$, $\epsilon_\gamma$ for counter No. 2, and to $\epsilon_1$, $\bar{\epsilon}_\gamma$ for counter No. 1. Since counter No. 1 does not record $\gamma$ rays, $\bar{\epsilon}_\gamma$ is the probability that a $\gamma$ ray emitted by a source in coincidence counting position will be absorbed in counter No. 1. Insertion of these values for $\epsilon_x'$, $\epsilon_\gamma'$ in Eqs. (B5) and (B6) gives

in counter No. 2:

$$R_{x+x} = (N'/2N) r_{xx}(\epsilon_2/\epsilon_1), \qquad R_{x+\gamma} = (N'/N) r_{x\gamma}(\epsilon_2/\epsilon_1); \tag{B7}$$

in counter No. 1:

$$R_{x+x} = (N'/2N) r_{xx}(\epsilon_1/\epsilon_2), \qquad R_{x+\gamma} = (N'/N) r_{x\gamma}(\bar{\epsilon}_\gamma/\epsilon_\gamma). \tag{B8}$$

Here good first approximations to $\epsilon_1$, $\epsilon_2$, $\epsilon_\gamma$ are, respectively, the ratios $R_1/x_t$, $R_2/x_t$, and $R\gamma/\Gamma$, where the $R$'s are those given in Eq. (2) of Sec. I and the values of $x_t$ and $\Gamma$, which are known essentially independently of the sum-pulse correction, are those given in Eq. (6) of Sec. I. For $\bar{\epsilon}_\gamma$, which cannot be computed in this way because the source was not $\gamma$-counted in counter No. 1, we make the very good assumption $\bar{\epsilon}_\gamma = (R_1/R_2)\epsilon_\gamma$. Starting with these first-approximation $\epsilon$'s and the uncorrected singles rates, accurate $\epsilon$'s and corrected singles rates for the coincidence-counting position can be obtained by iteration between Eqs. (B1), (B2), (B7), and (B8).

Furthermore, when these accurate $\epsilon$'s are obtained they can be inserted in Eqs. (B5) and (B6), and it then becomes a simple matter to make sum-pulse corrections on the singles rates of the auxiliary source (see Table II), since the relevant $\epsilon'$'s are known by computation.

An alternative method for correcting the $\gamma$-ray rates for sum pulses is to $\gamma$-count the source through sufficient absorber to suppress the x rays. We have tried this method in one case and found a corrected $\gamma$-ray rate that agreed within a percent with that obtained by the first method. However, the first method is the preferable one by far, because it also provides the sum-pulse corrections to the x-ray rates (which the absorber method cannot), requires no supplementary measurements or additional knowledge of x-ray attenuation coefficients, and is, we believe, more accurate, to say nothing of its being more elegant.